# An Efficient Volterra-Based Behavioral Model for Wideband RF Power Amplifiers

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*Abstract* — **Efficient and accurate behavioral modeling of RF power amplifiers with memory effects becomes of critical importance in the system-level analysis and design of wide band digital communication systems. In this paper, we present a novel Volterra-based behavioral model implemented through a bank of parallel FIR filters, the coefficients of which may be readily extracted from time-domain measurement or circuit envelope simulation. This model can reproduce the nonlinear distortion of power amplifiers with memory effects excited by wideband modulated signals with better accuracy compared to conventional quasi-memoryless models.**

## I. INTRODUCTION

In system-level simulation and design, behavioral models are often employed to predict the distortion created by nonlinear components. In these models, the nonlinear components are characterized in terms of input and output complex envelopes using relatively simple mathematical expressions. Behavioral modeling techniques provide a convenient and efficient means to predict system-level performance without the computational complexity of full circuit simulation or physical level analysis of nonlinear systems, thereby significantly speeding up the analysis process.

Behavioral modeling for RF power amplifiers has received much attention from many researchers in recent years. Most approaches are based on AM/AM and AM/PM models or polynomial memoryless models, or else suboptimal approximate systems. These methods are not sufficiently accurate for future wideband communication systems, especially with complex modulated signal systems. In recent decades, a truncated Volterra series model [1] has been used by a number of researchers to describe the relationship between the input and the output of a nonlinear system with memory. However, high computational complexity makes methods of this kind impractical for real-time implementation.

In this paper, we present a Volterra-based modeling technique for wideband RF power amplifiers, which utilizes novel concepts such as V-vector algebra [2] and multi-channel embedding [3] to implement a fast parallel nonlinear Volterra filtering algorithm [4][5]. This approach dramatically reduces computational complexity, and allows reproduction of both transient and steady-state behavior of RF power amplifiers with good accuracy.

The remainder of the paper is organized as follows. The principles of a new modeling technique are first outlined in section II. Then section III gives the model extraction methodology. Application the new behavioral model to a wideband LDMOS class AB power amplifier is described in section IV. Comparisons with the measurement results and conventional quasi-memoryless models are also given.

## II. MODEL PRINCIPLE

Well-established techniques for narrowband system level modeling (such as using AM/AM and AM/PM curves) can fail in the case of simulation of wideband signals because the output response of the power amplifier at a given instant depends not only on the input signal at the same time instant but also the input signal at preceding instants over a limited duration, leading to so-called "memory effects". A Volterra series resembles a Taylor series expansion, and can be used to represent a wide class of time-invariant nonlinear systems with memory effects. Consider $x(t)=\Re e\left[\tilde{X}(t)\cdot e^{j\omega_0 t}\right]$ and $y(t)=\Re e\left[\tilde{Y}(t)\cdot e^{j\omega_0 t}\right]$ as the input and output signal of a power amplifier, where $\omega_0$ is carrier frequency and $\tilde{X}(t)$ and $\tilde{Y}(t)$ represents the complex-valued envelopes of the input and output signal, respectively.

Using A/D conversion, a discrete time-domain finite-memory complex baseband Volterra model has the form:

$$
\begin{aligned}
\tilde{Y}(n) &= \sum_{i=0}^{m_1-1} h_1(i)\times\tilde{X}(n-i) \\
&+ \sum_{i_1=0}^{m_3-1}\sum_{i_2=i_1}^{m_3-1}\sum_{i_3=i_2}^{m_3-1} h_3(i_1,i_2,i_3)\times\tilde{X}(n-i_1)\tilde{X}(n-i_2)\tilde{X}^*(n-i_3) \\
&+ \sum_{i_1=0}^{m_5-1}\sum_{i_2=i_1}^{m_5-1}\sum_{i_3=i_2}^{m_5-1}\sum_{i_4=i_3}^{m_5-1}\sum_{i_5=i_4}^{m_5-1} h_5(i_1,i_2,i_3,i_4,i_5)\times\prod_{j}^{5}\tilde{X}(n-i_j)+\cdots+\eta(n)
\end{aligned}
\tag{1}
$$

where $h_l(i_1,i_2,\cdots i_l)$ is the $l$th-order Volterra kernel, $m_l$ represents the "memory" of the corresponding non-linearity, $(\bullet)^*$ represents the conjugate transpose and $\eta(n)$ is the unmeasured disturbance. In the above equation, we have removed the redundant items associated with kernel symmetry, and also the even-order kernels, whose effects can be omitted in band-limited modulation systems.

The outputs of a Volterra model are linear with respect to the kernels, and hence many fast least square (LS) adaptive algorithms for Volterra systems may be implemented using the same general procedure as would be employed for conventional linear digital filters with the exception that the input vectors must be expanded and redefined appropriately [6]. However, due to the loss of the time–shift property in the input data vector, direct application of linear adaptive algorithms to the Volterra case can significantly increase the computational complexity, which is a critical issue for many real-time applications. A novel non-rectangular structure matrix, shown in Fig. 1, termed the 'V-vector', has been developed [2] which can preserve the linear time-shift property for non-linear data vectors thereby avoiding the complex permutations that would otherwise be required. Recently, this has been used successfully for distortion compensation [7]. Using V-vector algebra, we can write (1) as

$$\tilde{Y}(n) = \hat{W}_n^H \tilde{X}_n \tag{2}$$

where $(\cdot)^H$ represents the Hermitian transpose of $(\cdot)$, while $\hat{W}_n$ and $\tilde{X}_n$ are the Volterra filter coefficients (kernels) and the input data V-vector, respectively.

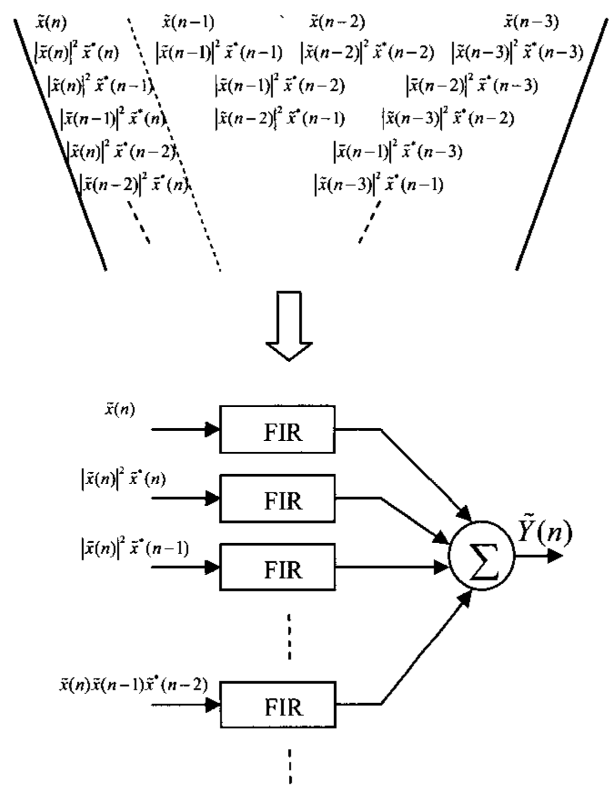


Fig. 1 A Volterra-Based Behavioral Model

Direct evaluation of the solution of (2) still requires a massive convolution at each instant [3]. However, using V-vector algebra, the new Volterra model inherits a time-shift invariance property (e.g. in every separate row of the input data vector $\tilde{X}_n$ in Fig. 1.) which allows implementation of the nonlinear system by a group of parallel linear sub-systems, such as transversal FIR filters. We can define a set of primary signals that carry all the information needed for the estimation of the convolution, corresponding to the first column of the input data V-vector $\tilde{X}_n$, and then use an FIR filter to implement the convolution for each row of $\tilde{X}_n$ separately. Summing together all the filter outputs, the final output of the Volterra behavioral model is obtained, as shown in Fig. 1. The primary signals are computed recursively from lower order products. This kind of parallel fast algorithm significantly improves the data processing speed and saves on computation time. Furthermore, this kind of time-domain envelope behavioral model may be readily embedded in most commercial CAD tools to reproduce the transient and static responses of power amplifiers.

## III. Model Extraction

The structure of the new Volterra behavioral model has been introduced in Section II. This section focuses on the model extraction methodology. The block diagram for model extraction is shown in Fig. 2. The complex envelope signals from the input and output of a power amplifier are fed into an adaptive filter to estimate the coefficients of the filters, which are in effect the Volterra kernels. When the system converges, the extraction process is finished. By copying the coefficients of the adaptive filter, the behavioral model is obtained. This type of modeling technique is nowadays easily carried out using envelope transient analysis in most CAD tools [8] or else using time-domain measurement [9][10].

By using V-vector algebra, the time-shift property of non-linear Volterra data vectors is preserved. Hence, in principle, any linear parameter estimation methodology can be utilized to extract the nonlinear Volterra kernel (the coefficients of the FIR filters in the behavioral model). However, in higher-order Volterra systems, directly updating the coefficient vector becomes very difficult even employing fast filtering algorithms because the matrix size of the coefficient and input data vectors increases quickly both with the order of the Volterra kernel and the memory length of the nonlinear system, and therefore the computational complexity required per iteration increases dramatically. In the section above, we have decomposed the Volterra model into a group of parallel transversal FIR filters. It is natural to utilize a parallel data processing technique [5] to update the coefficients of the filters during the learning period as well.

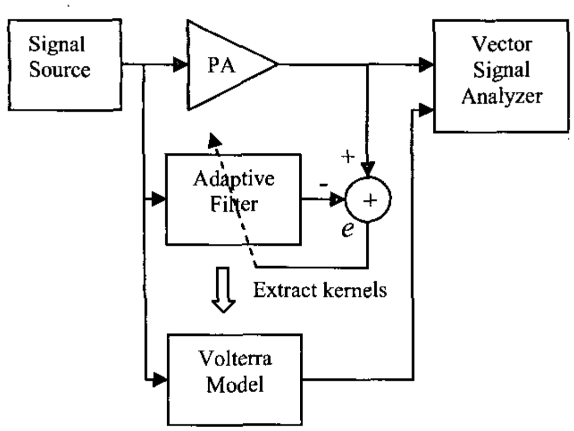


Fig. 2 Block Diagram for Model Extraction

In adaptive filters, the recursive least-square (RLS) algorithm is known to have very good convergence characteristic, and many fast RLS algorithms have been developed [4]. Generally, the objective of the exponentially-weighted RLS filter is to select the optimal coefficient $\hat{W}(n)$ in such a manner that the cost function defined by

$$J(n)=\sum_{k=1}^{n}\lambda^{n-k}(d(k)-\hat{W}^{H}(n)\tilde{X}(k))^{2} \tag{3}$$

is minimized at each time instant. In this formulation, $d(k)$ is the desired signal, and $\lambda$ is a constant that controls the speed of convergence of the adaptive filter and $0\le\lambda\le1$. The solution to the minimization problem in (3) can be found by differentiating $J(n)$ with respect to $\hat{W}(n)$, setting the derivatives to zero, and solving the resulting set of simultaneous equations. In a Volterra model with many kernels, direct updating of $\hat{W}(n)$ involves large matrix multiplications and inversions. However, partitioning $\hat{W}(n)$ and $\tilde{X}(n)$ into several smaller sub-vectors [5], e.g. one sub-vector for each order, the matrix size used for updating the filter coefficients becomes smaller, thereby reducing the computational complexity. Thus the problem is changed to one of minimizing $J_i(n)$ in every sub-filter. To minimize $J_i(n)$, we need to know the error signal $e_i(n)$ in the sub-filter. In a practical situation, we do not know $e_i(n)$ because the desired signal $d_i(n)$ in the sub-filter is not available. But we can compute the overall system output signal $\hat{Y}(n)$ by summing the outputs of all the sub-filters. We can then subtract $\hat{Y}(n)$ from $d(n)$ to get $e(n)$, and update all the sub-filters using $e(n)$. In other words, assume $e_i(n)=e(n)$ for all sub-filters. Then we can utilize a fast RLS filtering algorithm for every sub-filter to update their coefficients independently and concurrently, which significantly saves the computational time required per iteration. However, by replacing $e_i(n)$ by $e(n)$, the convergence rate of this parallel filtering algorithm will be slower compared with a direct use of the fast RLS algorithm. In real applications, we would need to trade off between the computational complexity and convergence speed required.

## IV. Model Validation

In order to validate the proposed model extraction methodology, the new model is tested by a LDMOS class AB medium power amplifier with noticeable memory effects at 2.14 GHz excited by downlink 3GPP W-CDMA signal of 3.84 Mcps chip rate and peak-to-average power ratio equal to 6.0 dB @ 0.01% probability on CCDF. The test bench setup uses the ADS-ESG-VSA connected solution from Agilent Technologies [10], shown in Fig. 3.

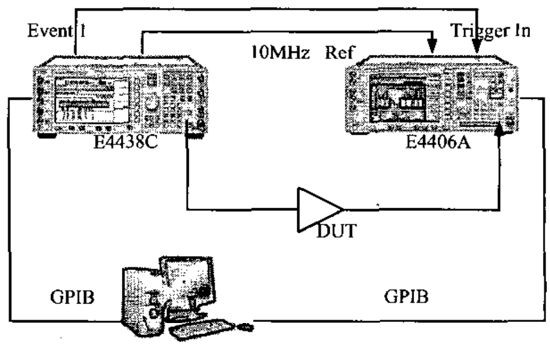


Fig. 3 The Experimental Test Bench

The baseband I/Q signals are generated from Agilent ADS software running on a PC, and downloaded to an Agilent E4438C ESG vector signal generator. This test signal is then passes through the DUT and into an Agilent E4406A vector signal analyzer (VSA). The DUT output test signal is then read from the E4406A VSA back into the ADS simulation environment using the Agilent 89601A VSA software, which is dynamically linked from within ADS. A 5th-order Volterra-based behavioral model of a power amplifier is extracted from the measurement input and output baseband complex envelope signals, and implemented in Matlab software from Mathworks Inc.

The time-domain output envelope waveform of W-CDMA signal is shown in Fig. 4. The average NMSE (*normalized mean square error*) [11] is up to -37dB, where an improvement of over 12 dB is gained by the new model, compared to the AM/AM AM/PM model. The frequency-domain spectra of the power amplifier output signal to W-CDMA excitation is shown in Fig. 5. The gain and ACPR performance of the power amplifier are given in Table I. Compared to the measurement results, the superior prediction of the amplifier nonlinear performance by the new model is clearly visible. A large improvement also has been made with respect to the classical AM/AM AM/PM model.

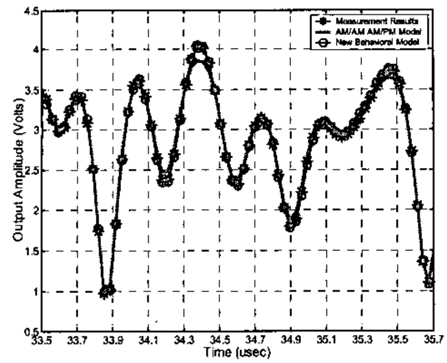

Fig. 4 Time-domain waveform amplitude of PA output with W-CDMA input signal

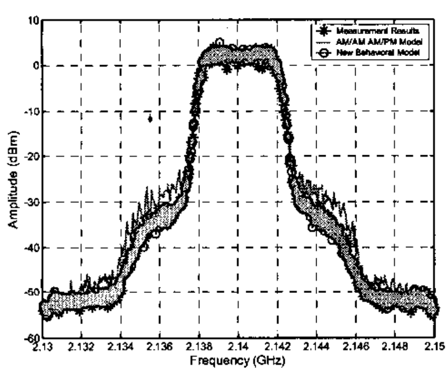

Fig. 5 Frequency-domain spectra of PA output with W-CDMA input signal

| Performance | Measurement Results | AM/AM AM/PM Model | New Behavioral Model |
|---|---|---|---|
| Gain (dB) | 13.19 | 13.16 | 13.17 |
| ACPR (dBC) (+/-5MHz) | 43.47/44.35 | 40.03/40.00 | 43.27/44.53 |
| ACPR (dBC) (+/-10MHz) | 56.52/57.38 | 54.66/54.62 | 56.44/57.00 |

Table I. Gain and ACPR of PA with 38dBm output

## V. CONCLUSIONS

An efficient Volterra-based modeling technique has been proposed. It can accurately reproduce nonlinear distortions of a power amplifier, including memory effects, allowing use of this modeling approach under wideband complex modulated signal applications. The extraction of the proposed model is simple and affordable either through circuit-level simulation or through calibrated time-domain envelope measurements. The model can also be readily embedded in most commercial CAD environments. Furthermore, the structure of this model is also naturally suited for baseband digital predistorter design [7] for high power amplifiers with memory effects since the roles of the input and output measurements can then be reversed, leading to a situation where the new model represents the inverse of the power amplifier's nonlinear characteristics.

## ACKNOWLEDGEMENT

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